

# Metamorphosis of the Cosmological Constant and 5D Origin of the Fiducial Metric

Gregory Gabadadze and Siqing Yu

*Center for Cosmology and Particle Physics, Department of Physics,  
New York University, New York, NY, 10003, USA*

## Abstract

In a recently proposed theory, the cosmological constant (CC) does not curve space-time in our universe, but instead gets absorbed into another universe endowed with its own dynamical metric, nonlocally coupled to ours. Thus, one achieves a long standing goal of removing entirely any cosmological constant from our universe. Dark energy then cannot be due to a cosmological constant, but must be obtained via other mechanisms. Here we focus on the scenario in which dark energy is due to massive gravity and its extensions. We show how the metric of the other universe, that absorbs our CC, also gives rise to the fiducial metric known to be necessary for the diffeomorphism invariant formulation of massive gravity. This is achieved in a framework where the other universe is described by 5D AdS gravity, while our universe lives on its boundary and is endowed with dynamical massive gravity. A non-dynamical pullback of the bulk AdS metric acts as the fiducial metric for massive gravity on the boundary. This framework also removes a difficulty caused by the quantum strongly coupled behavior of massive gravity at the  $\Lambda_3$  scale: in the present approach, the massive gravity action does not receive any loop-induced counterterms, despite being strongly coupled.

# 1 An Unconventional Path

The importance of the cosmological constant (CC) problem is well-known, so are the difficulties in solving it (see [1, 2], and references therein). It is clear that unconventional approaches are needed. Along these lines, Tseytlin [3] had made an interesting proposal building on an earlier idea of [4], and inspirations from T-duality of string theory. He suggested to apply the least action principle to the “volume normalized action,”  $\bar{S}$ :

$$\bar{S} \equiv \frac{S}{V_g} = \frac{\int d^4x \sqrt{|g|} (R + 2\mathcal{L}_{\text{SM}})}{\int d^4x \sqrt{|g|}}, \quad (1.1)$$

instead of that principle being applied to  $S$ . Here,  $V_g = \int d^4x \sqrt{|g|}$  is the invariant space-time volume in the  $16\pi G_N = 1$  units (it is assumed that  $V_g$  is regularized, as in Section 5). The above modified action should be thought as a certain low energy effective action, hopefully emerging from more conventional high energy physics [3]. Since the CC problem is a low energy problem, it is reasonable to expect that its solution will not be influenced by an exact form of the high energy completion of (1.1).

Furthermore,  $\mathcal{L}_{\text{SM}}$  in (1.1) denotes a Lagrangian of all the fields of nature but gravity, coupled to gravity. As argued in [5], for consistency with the empirical data the Lagrangian  $\mathcal{L}_{\text{SM}}$  needs to be regarded as a quantum effective Lagrangian coupled to classical gravity. This classical gravity will be subsequently quantized using (1.1). Such an unconventional procedure of quantization will be reviewed in Section 6, where earlier works that made this procedure more precise will be referenced. Till then we will not use any specific form of  $\mathcal{L}_{\text{SM}}$ , but discuss only its constant part.

It is straightforward to see that a cosmological constant is an unphysical parameter in (1.1): a shift of  $\mathcal{L}_{\text{SM}}$  by any constant changes  $\bar{S}$  by an additive constant, and the latter does not affect the equations of motion obtained by varying  $\bar{S}$ .

While this appears to be an efficient way to get rid of an arbitrary CC irrespective of its origin, the equations of motion obtained from  $\bar{S}$  reveal the high cost of the proposed solution: in the equations, the local Ricci curvatures are determined by spacetime averages over past and future volumes, when these volumes are well-defined [3]. This seems to be a dramatic departure away from the conventional local field theory paradigm.

Such spacetime nonlocality (and therefore, acausality [6]) is however operative only for a cosmological constant, while all the local interactions that can be measured in a laboratory – even if in one with the size of the observed universe – are not affected. One may wonder how the mechanism could distinguish between a cosmological constant and, say, a very flat scalar potential. It does so by being nonlocal in time, and hence being able to target and eliminate only the cosmic fluid that does not red-shift in future infinity [6]. As a result, a CC is eliminated, while the slow roll inflation, radiation, and matter dominated epochs of cosmology remain intact.<sup>1</sup>

What about non-cosmological solutions of Einstein’s gravity? Since the spatial nonlocality of the proposed equations is operative only for sources that have an infinite spatial

---

<sup>1</sup>The mechanism is applicable to a single vacuum universe [3, 4]. Its generalization to the case that adopts chaotic inflation with multiple vacua is not known to the authors. Throughout the paper we focus on a scenario in which our 4D universe evolves in a single vacuum state.

support, then intact remain also all spatially local solutions of Einstein's gravity. Thus, the high cost incurred by the solution of the CC problem, while somewhat bothersome, remains undemanding, at least until inconsistencies of the proposal are found.

We add that the action (1.1) can readily be used to obtain the classical Hamiltonian of the theory: The multiplier,  $1/V_g$ , does not contain any derivatives of the metric, and affects the determination of the canonical momenta only by rescaling them with a multiplicative numerical factor of  $1/V_g$ .

To summarize, in the context of classical gravity and conventionally quantized particle physics, if  $\bar{S}$  is used as a gravitational action for a single vacuum state in which the universe is presumed to evolve, then the CC is removed, without any apparent contradiction with observations (see also [7], for further independent work on the proposal of [3]).

However, the trouble comes with the graviton quantum loops [3]. This problem emerges in the low energy theory, even before we ask the question of the existence of a full-fledged UV complete quantum theory of gravity, from which the action (1.1) might originate. To see this, we recall that the quantization of (1.1) would use the path integral with the kernel

$$\exp\left(i\frac{\bar{S}}{\hbar}\right) \equiv \int_{-\infty}^{+\infty} d\lambda d\tau \exp\left\{\frac{i}{\hbar}\left[\frac{1}{\tau}S + \lambda(V_g - \tau)\right]\right\}, \quad (1.2)$$

where we have introduced integrations w.r.t.  $\tau$  and  $\lambda$ , which are real parameters.<sup>2</sup> It is clear from (1.2) that the new Planck constant that governs the loop expansion for gravity is  $\hbar\tau$ . This quantity is infinite (or at least very large in units of the Planck length), since  $\tau = V_g$  is infinite (or at least as large and old as the present-day universe); therefore, the loop expansion in  $\hbar\tau$  would diverge and ruin the aforementioned classical solution of the big CC problem [3]. Even if a resummation of the loop expansion were found, the result would significantly differ from (1.1), and one would have to work anew for a solution of the CC problem. There is no reason to expect that the would-be resummed quantum action would retain the property of (1.1) which made a CC unphysical.

## 2 Beyond the Unconventional Path

A proposal was put forward in [5] that appears to retain the good part of Tseytlin's approach, but gets rid of the difficulty with the gravity loops. This is achieved by introducing a second universe, endowed with its own metric  $f$ , that interacts with our universe only globally via the following action:

$$\mathcal{A} = V_f \bar{S} + S_f. \quad (2.1)$$

Here,  $\bar{S}$  is defined in (1.1), and is a functional of the metric  $g$  and the SM fields. The action  $S_f$  contains the Einstein-Hilbert (EH) term for  $f$ , but is independent of the metric of our universe,  $g$ . From the point of view of the  $f$ -universe, the term  $V_f \bar{S}$  represents an addition to a CC in that universe, since  $V_f$  is its invariant volume.

Thus, it is straightforward to see from (2.1) that an arbitrary cosmological constant introduced in  $S$  – the numerator in (1.1) – becomes a cosmological constant of the  $f$ -universe.

---

<sup>2</sup>The  $\lambda$  integration is needed to determine  $\tau$ , while the  $\tau$  integration determines  $\lambda$ ; the latter enters the metric equations of motion.

Therefore, there is no big CC problem in the  $g$ -universe – the CC of the  $g$ -universe gets entirely absorbed into the  $f$ -universe [5].

We note that the action (2.1) can be readily used to obtain the Hamiltonians for both  $g$ - and  $f$ - universes. The unusual feature of (2.1) is the multiplier  $V_f/V_g$ . The latter contains no derivatives of the metrics. Thus, this multiplier affects the determination of the canonical momenta of the  $g$ -universe only by a factor of  $V_f/V_g$ . This however does not represent an impediment for writing down the classical Hamiltonian of the theory.

How about quantum corrections? From the arguments presented after (1.2), it should be evident that the quantum loop effects will be small as long as  $V_g/V_f \lesssim 1$ . Let us elaborate on this. The kernel of the path integral for (2.1) can be written as follows,

$$\exp\left(i\frac{\mathcal{A}}{\hbar}\right) = \int d\lambda dq \exp\left\{\frac{i}{\hbar}\left[\frac{1}{q}S + \lambda\left(\frac{V_g}{V_f} - q\right) + S_f\right]\right\}. \quad (2.2)$$

Then, the new Planck constant for quantization of  $g$ -gravity is  $\hbar q$ , which is now equal to  $\hbar(V_g/V_f)$ . As long as there are self-consistent classical solutions for  $g$ - and  $f$ - metrics such that  $V_g/V_f \ll 1$ , the quantum loop corrections to the classical action will be negligible.<sup>3</sup>

Hence, the main remaining task is to arrange for the dynamics of the  $f$ -universe to satisfy  $V_g/V_f \ll 1$ , in a technically natural way. This was achieved in [5] by postulating that  $S_f$  is the action of 4D Einstein's gravity with a negative cosmological constant, which is somewhat larger than the CC generated in the  $g$ -universe. If so, then the CC of the  $g$ -universe gets entirely absorbed into the  $f$ -universe, and since its magnitude is smaller than that of the CC in the  $f$ -universe, it only modifies the  $f$ -metric slightly. As a result, there is a classical solution for which the metric of the  $f$ -universe is  $\text{AdS}_4$ , while the metric of the  $g$ -universe is flat!

However, we cannot stop here since we need to describe the cosmic acceleration. This should be done by means other than using a CC. There could be a few options here.<sup>4</sup> We choose to introduce dark energy via massive gravity [8, 9], or its extensions.<sup>5</sup> This theory postulates a small graviton mass,  $m \sim 10^{-33}$  eV. Albeit the small parameter put in by hand, this is a technically natural way of introducing dark energy, since the graviton mass, unlike a CC, does not receive the additive power-divergent quantum corrections, and its renormalizations are multiplicative [13].

Once the dark energy is introduced, the  $g$ -metric would turn into that of a  $\text{dS}_4$  spacetime with a tiny observable curvature. Our mechanism will then be at work since the invariant volume of the universal cover of  $\text{AdS}_4$  is infinitely larger than the infinite volume of  $\text{dS}_4$ . Thus, we have the desired result,  $V_g/V_f \rightarrow 0$ , and the classical solutions described above remain self-consistent even after the quantum gravity loops are taken into account [5].

We note that the above proposal does not eliminate the motivation for quantum gravity. The latter would be needed to obtain the action  $\mathcal{A}$  from more fundamental theory, and also to quantize the action  $S_f$  in (2.1). Furthermore, the mechanism does not exclude higher

---

<sup>3</sup>One should also worry about: (a) new nonlocal corrections that would arise in (2.1), and (b) new loop-generated terms containing polynomials of  $\lambda$  and  $q$  in (2.2). Neither (a) nor (b) will cause problems, as to be addressed in Section 6.

<sup>4</sup>For instance, a 4D quintessence model can be used to get dark energy in the framework described in Sections 3 and 4 of the present paper.

<sup>5</sup>See reviews [10, 11, 12] on theory and phenomenology of massive gravity and its extensions.

powers of the curvature terms and their derivatives in  $S$ , if they arise due to the  $\alpha'$  expansion of a putative completion into string theory. In fact, an infinite number of such terms in  $S$  would be welcome in the present framework – the black hole and cosmological singularities could hopefully be resolved by these  $\alpha'$  terms (see more in Section 6).

### 3 A New Embedding

The construction of [5] invoked two assumptions: (I) the CC in the  $f$ -universe was assumed to be large and negative, without tunings of any kind; (II) dark energy in the  $g$ -universe was postulated to be due to massive gravity. Both assumptions seem to be of a provisional character, as one could come up with other scenarios giving  $V_g/V_f \ll 1$ , with dark energy produced by a different technically natural mechanism.

Having said this, however, we note that assumption (I) may have a strong justification: the  $f$ -universe with its negative CC could be supersymmetric, and broken supersymmetry in the  $g$ -universe needs not be communicated to the  $f$ -universe, since  $f$  and  $g$  are only globally connected [5].

How about assumption (II)? It is not necessary, but could there be advantages in using massive gravity, as opposed to other forms of dark energy (CC excluded)? Are there specific features of massive gravity that naturally fit into the above proposal for solving the big CC problem? If the answers to these questions were positive, then the proposed mechanism for the CC problem would be connected to the mechanism for dark energy.

In the present work, we claim partial success in this quest. In particular, we will show how the fiducial metric written by means of the Stückelberg fields – that are necessary for the diffeomorphism invariant formulation of massive gravity – can be related to the embedding and metric of the 5D  $f$ -universe, introduced to solve the big CC problem. Our construction unveils a higher dimensional origin of the fiducial metric in massive gravity, and gives more appeal to the scenario in which dark energy is due to massive gravity and its extensions.

As a by-product, we will show that embedding of massive gravity into the action  $S$  in (2.1) helps deal with a well-known difficulty of this theory related to its quantum strong coupling behavior at the scale  $\Lambda_3 = (M_{\text{Pl}} m^2)^{1/3}$ : the quantum loop corrections to the massive gravity action (and those in its extensions) are now controlled by  $\hbar q \rightarrow 0$ . Therefore, the loop diagrams do not generate any counterterms, beyond the terms already present in the tree-level action. This does not eliminate the scale  $\Lambda_3$ , and the theory is still strongly coupled at that scale. However, the effective action receives no loop-generated counterterms.<sup>6</sup> Thus, the full quantum effective action has as few terms as the tree-level action, and calculations done in this theory – albeit strongly coupled at  $\Lambda_3$  – are exact, modulo the high derivative curvature terms that presumably appear at the Planck scale ( $M_{\text{Pl}} \gg \Lambda_3$ ) as part of the completion at that scale. This is unlike a generic non-renormalizable theory that would generate an infinite number of new counterterms at the scale  $\Lambda_3$ .

The main idea of the present work is to regard the action of the “other” universe as the EH action of 5D gravity, described by the metric  $F$  on  $\text{AdS}_5$ . Hence, we use the notation  $S_F$ , instead of  $S_f$  used previously, to indicate that the “other” universe is five-dimensional; we denote its invariant volume by  $V_F$ . We then endow a hypersurface at the  $\text{AdS}_5$  boundary

---

<sup>6</sup>For detailed discussions of the loops and optical theorem in this framework, see Section 6.

with the dynamical metric  $g$ , and couple  $g$  to the pullback of the bulk metric,  $\gamma$ , in such a way that the latter acts as the fiducial metric of 4D massive gravity described by  $g$ .

The pullback  $\gamma$  can be expressed via the four Stückelberg scalars of massive gravity  $\varphi^a(x)$ ,  $\gamma_{\mu\nu} = \partial_\mu \varphi^a \partial_\nu \varphi^b \eta_{ab}$  ( $a, b, \dots = 0, 1, 2, 3$ ). These scalars then end up parametrizing the 4D hypersurface at the boundary of the  $\text{AdS}_5$  bulk. The detailed construction is worked out in Section 4, where a simple local model without the  $V_F/V_g$  factor is considered. Besides its illustrative purpose, this model also delineates a method of introducing gravity, albeit massive, on the boundary of  $\text{AdS}_5$ . The full model (2.1) is examined in Section 5, where we show how the fiducial metric of massive gravity emerges from the pullback of the bulk metric in  $\text{AdS}_5$ . The discussions of the quantum loop effects and the strong coupling problem of massive gravity in this approach are given in Sections 6 and 7, respectively. Section 8 contains comments and outlook.

## 4 Massive Gravity On Top of the Boundary of $\text{AdS}_5$

Our bottom-up construction begins with postulating a dynamical 4D metric,  $g_{\mu\nu}$ , on top of the boundary of  $\text{AdS}_5$ . The metric  $g$  is not related to a pullback of the bulk gravity. Hence, such a boundary has more structure than the conventional conformal boundary of  $\text{AdS}_5$ , and may be thought as some geometric hypersurface, that is endowed with  $g$  and is placed right at the boundary of  $\text{AdS}_5$ .

To set the conventions,  $x^\mu$  (with  $\mu, \nu, \dots = 0, 1, 2, 3$ ) denote the coordinates of 4D space-time of the hypersurface that is endowed with the dynamical metric  $g_{\mu\nu}(x)$ . The massive gravity action defined via  $g$  on the world-volume of the boundary hypersurface reads as follows,

$$S = M_{\text{Pl}}^2 \int d^4x \sqrt{|g|} [R(g) - 2\Lambda + 2m^2 \mathcal{U}(\mathcal{K})], \quad (4.1)$$

where the diff-invariant potential  $\mathcal{U}$  was built in [8, 9], as a function of the inverse metric  $g^{-1}$ , and the fiducial Minkowski metric,  $\gamma_{\mu\nu} = \partial_\mu \varphi^a \partial_\nu \varphi^b \eta_{ab}$  ( $a, b, \dots = 0, 1, 2, 3$ ), in an arbitrary coordinate system; this potential can be written in the following form:

$$\mathcal{U}(\mathcal{K}) = \det_2(\mathcal{K}) + \alpha_3 \det_3(\mathcal{K}) + \alpha_4 \det_4(\mathcal{K}), \quad (4.2)$$

where the matrix  $\mathcal{K} = 1 - A$ , with  $A$  being defined as one of the roots of,  $A_\alpha^\mu A_\nu^\alpha = g^{\mu\alpha} \gamma_{\alpha\nu}$ , so that  $\mathcal{K} = 1 - \sqrt{g^{-1}\gamma}$  [9].

The fiducial Minkowski metric,  $\gamma$ , is not dynamical, and its origin is unknown in massive gravity, because the dynamical origin of the fields  $\varphi^a$  is not known. In the high energy limit, these fields parametrize the helicity  $\pm 1$  and helicity 0 degrees of freedom of a massive graviton. Geometrically, they can be regarded as coordinates of a certain fiducial 4D Minkowski space, that is postulated as "pre-geometry" in pure massive gravity (see discussions in [14]).<sup>7</sup>

---

<sup>7</sup>Note that the origin of the fields  $\varphi^a$  is not elucidated in conventional 4D bigravity theories either for the following reasons: these theories have two dynamical metrics,  $g(x)$  and  $\tilde{g}(x)$ , however only one common diffeomorphism for the two; to restore the second diffeomorphism in bigravity, one has to introduce the four fields,  $\varphi^a$ , and postulate that the second metric is a function of these fields,  $\tilde{g}(\varphi)$ . While the diff invariance, as any local gauge symmetry, is a redundancy of description, it is a helpful redundancy, and in the case

While this work does not provide the Higgs mechanism for gravity, it supplies geometric meaning for the fields  $\varphi^a$ . This is done in the present approach by relating the fiducial metric to the pullback of the bulk metric, where  $\varphi^a$ 's get related to 5D coordinates. To see this relation we denote the bulk coordinates in  $\text{AdS}_5$  as  $Y^A$ 's with  $A, B, \dots = 0, 1, 2, 3, 5$ , and relate the 5D and 4D coordinates as follows,

$$Y^A = \{\varphi^a(x), z\}. \quad (4.3)$$

In this coordinate system, the 4D boundary hypersurface is located at  $z = 0$ , while the fields  $\varphi^a(x)$  (with  $a, b, \dots = 0, 1, 2, 3$ ) give different parametrizations of the hypersurface. In other words, these fields map the 4D spacetime on which the  $g(x)$  metric lives, onto the 4D hypersurface in 5D  $\text{AdS}$  spacetime.

As to the bulk action, it is the 5D EH action with the negative CC and the Gibbons-Hawking terms included (see, e.g., [22]),

$$S_F = \int_M d^5Y \sqrt{|F|} \left( R(F) + \frac{12}{l^2} \right) + 2 \int_{\partial M} d^4Y \sqrt{|\tilde{\gamma}|} K - \frac{6}{l} \int_{\partial M} d^4Y \sqrt{|\tilde{\gamma}|}. \quad (4.4)$$

Here, the 5D Planck mass  $M_5$  is set to unity,  $\tilde{\gamma}$  is the induced metric on the boundary, and  $K$  is the trace of the boundary extrinsic curvature. Our ultimate goal is to study the functional

$$\mathcal{A}_5 = \frac{V_F}{V_g} S + S_F, \quad (4.5)$$

but in this section, as a warm-up exercise, we study (4.5) without the  $V_F/V_g$  factor in front of  $S$ . Thus, the total action of the theory considered in this section is

$$S_{\text{tot}} = S + S_F. \quad (4.6)$$

The action (4.4) contains a negative cosmological constant,  $-\frac{6}{l^2}$ , and thus generates an  $\text{AdS}_5$

---

of massive gravity and bigravity, it calls for understanding of the dynamical origin of the helicity  $\pm 1$  and helicity 0 degrees of freedom that are encoded in  $\varphi^a$ . The lack of this understanding is the reason why both massive gravity and bigravity in the existing formulation are strongly coupled theories at an energy scale that is much lower than the Planck scale. The above comments can be briefly summarized as follows: the analog of the Lorentz-invariant Higgs mechanism for gravity or bigravity – from which  $\varphi^a$ 's would originate – is not known at present.

Before we proceed, we make a few comments on the literature: A partial list of the earlier bigravity theories is in [15, 16]. That the problem, now referred as the Boulware-Deser (BD) ghost problem, potentially applies to all massive gravity and bigravity theories was shown in Ref. [17]. After the terms (4.2) were proposed in Ref. [9] as ghost free terms, proven to be free of the BD ghost up to the fourth order in nonlinearities [9], and proven to be BD ghost free to all orders in [18], these terms were invoked by Ref. [19] into the earlier bigravity theories [16] and it was shown that the resulting theories are free of the BD ghost [19]. This prove, and notably the works [20] and [21], imply that the earlier versions of bigravity [15, 16] – except two bigravity models by Wess and Zumino (WZ) in [15] – suffer from the BD ghost. Two out of the four WZ models introduced the subset of terms (4.2), written in the first order formalism for two tetrads, albeit the WZ work does not address the nonlinear consistency and the BD problem, since it precedes the BD work by two years or so.

solution to the respective Einstein equations,

$$\begin{aligned}
ds^2 &= F_{AB}^{(0)} dY^A dY^B \\
&= \frac{l^2}{z^2} (\eta_{ab} d\varphi^a d\varphi^b + dz^2) \\
&= \frac{l^2}{z^2} (\eta_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b dx^\mu dx^\nu + dz^2).
\end{aligned} \tag{4.7}$$

The interval in the last line is written in terms of the boundary massive gravity coordinates.

We are now in a position to relate the induced metric on the boundary,  $\tilde{\gamma}$ , to the fiducial metric of massive gravity,  $\gamma$ . Our choice for this relation reads as

$$\gamma_{\mu\nu}(x) = \lim_{z \rightarrow 0} \frac{z^2}{l^2} \tilde{\gamma}_{ab}(z, \varphi(x)) \frac{\partial \varphi^a}{\partial x^\mu} \frac{\partial \varphi^b}{\partial x^\nu}. \tag{4.8}$$

This identifies the Stükelberg fields used in the massive gravity potential as the coordinates of the boundary in the  $\text{AdS}_5$  bulk. The variational procedure for the total action is defined as follows:  $S_F$  is varied w.r.t.  $F$ . The boundary action,  $S$ , on the other hand, is varied w.r.t.  $g$ , but no variation is taken w.r.t.  $F$ . The induced bulk metric on the boundary,  $\tilde{\gamma}$ , and the fiducial metric,  $\gamma$ , are then related to each other via the boundary condition (4.8). This procedure is compatible with the variational principle in  $\text{AdS/CFT}$ , since varying the boundary fields does not affect the bulk dynamics. Under this premise, the linearized theory with Dirichlet boundary data is studied next.<sup>8</sup>

The derivation of the on-shell boundary effective action from the bulk action,  $S_F$ , is a well-known calculation in standard  $\text{AdS/CFT}$ . Here we give an outline of this procedure adapted to our action (4.6). For details, we refer the readers to Appendix A.

In the following, we will work at the regularized boundary  $z = \epsilon$ , and the limit  $\epsilon \rightarrow 0$  will be taken only at the end of the calculations. In the linearized theory, the full bulk metric is  $F_{AB} = F_{AB}^{(0)} + h_{AB}$ , where  $h_{AB}$  denote the components of 5D metric fluctuations. We work in the gauge  $h_{0B} = 0$  and introduce  $h_{ab}(z, \varphi) = z^2 h_{AB} \delta_a^A \delta_b^B / l^2$ . Then we can write

$$ds^2 = \frac{l^2}{z^2} [(\eta_{ab} + h_{ab}) d\varphi^a d\varphi^b + dz^2]. \tag{4.9}$$

By (4.8), the fiducial metric on the boundary receives corrections from the 5D fluctuations,

$$\gamma_{\mu\nu}(x) = \eta_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b + \delta\gamma_{\mu\nu}(x), \tag{4.10}$$

where

$$\delta\gamma_{\mu\nu}(x) = \lim_{\epsilon \rightarrow 0} h_{ab}(\epsilon, \varphi(x)) \partial_\mu \varphi^a \partial_\nu \varphi^b. \tag{4.11}$$

Thus,  $\delta\gamma_{\mu\nu}$  represent the induced fluctuations. We also define the related metric  $\gamma_{ab}(\varphi(x)) = \eta_{ab} + h_{ab}(\epsilon, \varphi(x))|_{\epsilon \rightarrow 0}$ , so that  $\gamma_{\mu\nu}(x) = \gamma_{ab}(\varphi(x)) \partial_\mu \varphi^a \partial_\nu \varphi^b$ . The resulting effective boundary action can be written as a sum of various terms

$$S[h_{ab}] = S_{\partial^2} + S_{\partial^4} + S_{\text{nonlocal}}, \tag{4.12}$$

---

<sup>8</sup>We use  $S_F$  as the bulk action even though it contains the Gibbons-Hawking boundary terms, the sole role of which is to guarantee the right bulk Einstein equations. In contrast, the action  $S$ , referred here as the boundary action, gives rise to the dynamical equations of motion on the boundary. We hope this nomenclature will not cause any serious confusion.



where the respective parts of the total action are defined as follows:

$$S_{\partial^2} \sim \frac{1}{\epsilon^2} \int d^4 \varrho \, h \partial^2 h, \quad S_{\partial^4} \sim \ln \epsilon \int d^4 \varrho \, h \partial^4 h, \quad (4.13)$$

$$S_{\text{nonlocal}} \sim \int d^4 \varrho d^4 \varsigma \frac{h(\epsilon, \varrho) h(\epsilon, \varsigma)}{|\varrho - \varsigma|^8}. \quad (4.14)$$

The exact formulae and tensor structures can be found in (A.21) – (A.24). In particular, the nonlocal terms give the two-point function for the boundary CFT stress-tensor,

$$S_{\text{nonlocal}} = -\frac{1}{2} \int d^4 \varrho d^4 \varsigma \, h_{ab}(\epsilon, \varrho) h_{cd}(\epsilon, \varsigma) \langle \mathcal{T}^{ab}(\varrho) \mathcal{T}^{cd}(\varsigma) \rangle, \quad (4.15)$$

$$\begin{aligned} \langle \mathcal{T}^{ab}(\varrho) \mathcal{T}^{cd}(\varsigma) \rangle &\equiv \frac{20}{\pi^2} \frac{1}{|\varrho - \varsigma|^8} \\ &\times \left[ \frac{1}{2} J^{ad}(\varrho - \varsigma) J^{bc}(\varrho - \varsigma) + \frac{1}{2} J^{ac}(\varrho - \varsigma) J^{bd}(\varrho - \varsigma) - \frac{1}{4} \eta^{ab} \eta^{cd} \right], \\ J^{ab}(\varrho) &= \eta^{ab} - 2 \frac{\varrho^a \varrho^b}{|\varrho|^2}. \end{aligned} \quad (4.16)$$

As a consistency check, the  $S_{\partial^2}$  terms comprise the standard kinetic terms for linearized Einstein gravity,

$$S_{\text{kin}}[h_{ab}] = \frac{1}{2\epsilon^2} \int d^4 \varrho \left[ \frac{1}{4} h_{ab} \square h^{ab} - \frac{1}{2} h_{ab} \partial^a \partial_c h^{bc} - \frac{1}{4} h \square h + \frac{1}{2} h_{ab} \partial^a \partial^b h \right]. \quad (4.17)$$

The local terms in (4.13) with divergent powers of  $\epsilon$  are cancelled by appropriate counter-terms introduced as part of the renormalization procedure. This procedure deprives the fields  $h_{ab}(\epsilon, \varphi)$  of any dynamics. As a result, one regards them as the Dirichlet data supplied by boundary sources. We highlight this fact by using the notation  $h_{ab}^0(\varphi(x)) = h_{ab}(\epsilon, \varphi(x))|_{\epsilon \rightarrow 0}$ , so that  $\gamma_{ab}(\varphi(x)) = \eta_{ab} + h_{ab}^0(\varphi(x))$ . Then the total effective boundary action is

$$\begin{aligned} S_{\text{tot}}^{\text{eff}}[g, \gamma] &= M_{\text{Pl}}^2 \int d^4 x \sqrt{|g|} [R(g) - 2\Lambda + 2m^2 \mathcal{U}(\mathcal{K})] \\ &\quad - \frac{1}{2} \int d^4 \varrho d^4 \varsigma \, h_{ab}^0(\varrho) h_{cd}^0(\varsigma) \langle \mathcal{T}^{ab}(\varrho) \mathcal{T}^{cd}(\varsigma) \rangle + \mathcal{O}((h^0)^3), \end{aligned} \quad (4.18)$$

where  $\mathcal{K} = 1 - \sqrt{g^{-1} \gamma_{ab}(\varphi(x)) \partial_\mu \varphi^a \partial_\nu \varphi^b}$ , contains a general fiducial metric [19, 23]; formally, the theory with  $h^0 \neq 0$  differs from massive gravity with a general fiducial metric discussed in [19, 23] by an infinite number of new polynomial terms in  $h^0$ , and can be regarded as a theory in some external background field set by  $h^0$ . However, these new polynomial terms do not enter the massive gravity equations of motion since they do not depend on either  $g$  or  $\varphi^a$ . Hence, when viewed as a 4D theory, the obtained model is nothing but massive gravity with an arbitrary fiducial metric parametrized by  $h^0$  [19, 23]. However, this is a limited view since the terms containing higher powers in  $h_0$  encode additional useful information:  $h^0$  can be regarded as an external source for a dual 4D field theory of the 5D AdS gravity, and if

so, the polynomials in  $h^0$  define dual CFT correlators:  $n$ th variation of the action (4.18) w.r.t.  $h^0$ , with the subsequent substitution  $h^0 = 0$ , calculates the CFT  $n$ -point correlation function, modified by the terms that are proportional to powers of the graviton mass. Thus, for a small graviton mass, one would arrive at a softly broken CFT. To summarize, our prescription is to put  $h^0 = 0$  after all the calculations are done. On the gravity side of the dual pair, this condition is a choice of the boundary values. On the CFT side, this choice – imposed after all the variations are done – enables us to get the CFT correlation functions.

## 5 Removing CC and Introducing Dark Energy

We now turn to the action functional that in the present context can remove an arbitrary 4D CC from the dynamical 4D boundary of 5D AdS gravity. In the spirit of (2.1), we look at the action (4.5),

$$\mathcal{A}_5 = \frac{V_F}{V_g} S + S_F. \quad (5.1)$$

Here,  $V_F = \int d^5 Y \sqrt{|F|}$  and  $V_g = \int d^4 x \sqrt{|g|}$ . All the conventions are the same as in the previous sections. To avoid ambiguity in the equations of motion due to ratios of infinite volumes, we regularize all integrals in (5.1),

$$\begin{aligned} \int d^4 x &= \lim_{\bar{x}^0, \dots, \bar{x}^3 \rightarrow \infty} \left( \prod_{\mu=0}^3 \int_{-\bar{x}^\mu}^{\bar{x}^\mu} \right) d^4 x \equiv \lim_{\bar{x} \rightarrow \infty} \int_{\text{reg}} d^4 x, \\ \int d^5 Y &= \int dz d^4 \varphi = \lim_{\bar{\varphi}^0, \dots, \bar{\varphi}^3 \rightarrow \infty} \lim_{\epsilon \rightarrow 0} \int_{\epsilon}^{\infty} dz \left( \prod_{A=0}^3 \int_{-\bar{\varphi}^A}^{\bar{\varphi}^A} \right) d^4 \varphi \equiv \lim_{\bar{Y} \rightarrow \infty} \lim_{\epsilon \rightarrow 0} \int_{\text{reg}} d^5 Y, \end{aligned} \quad (5.2)$$

so (5.1) is actually

$$\mathcal{A}_5 = \lim_{\bar{x}, \bar{Y} \rightarrow \infty} \lim_{\epsilon \rightarrow 0} \left[ \frac{V_F}{V_g} S + S_F \right]_{\text{reg}}. \quad (5.3)$$

The order of the limits is carefully arranged:  $\epsilon \rightarrow 0$  is taken before  $\bar{x}, \bar{Y} \rightarrow \infty$ . This regularization first restricts the theory to a 5D “box,” then extends the  $z$  direction to include the conformal boundary, before taking the remaining limits. For conciseness, we will always treat the volumes as regularized, but will not write the regularizations explicitly.

It is clear that in the action (5.1), the 4D CC does not curve the 4D spacetime, but instead gets absorbed into the 5D CC. If the magnitude of the 5D CC is greater than that of the 4D CC, then this leads to an obvious modification of the curvature of the AdS bulk, while all the conclusions of the previous section remain valid. An order-of-magnitude hierarchy between the scales of the 5D and 4D CC’s is straightforward to arrange, since 5D theory can be exactly supersymmetric, while the broken SUSY in 4D could guarantee its scale to be lower.<sup>9</sup> Moreover, since  $V_F$  now is the five-volume of the 5D AdS spacetime, and  $V_g$  is going to be the four-volume of the 4D dS spacetime, the ratio  $V_g/V_F \rightarrow 0$ , in Planck units. This guarantees that the outlined classical solution is stable w.r.t. quantum loops.

---

<sup>9</sup>For similar arguments, see [5].

What remains to be seen is how the fiducial metric arises in the boundary massive gravity, and how the theory gives rise to dark energy. To this end, we just repeat, step by step, the procedure of “integrating out” the AdS bulk described in the preceding section. The result, in the quadratic order in the bulk, reads as follows:

$$S_{\text{tot}}^{\text{eff}}[g, \gamma] = \frac{V_F}{V_g} M_{\text{Pl}}^2 \int d^4x \sqrt{|g|} [R(g) + 2m^2 \mathcal{U}(\mathcal{K})] - \frac{1}{2} \int d^4\varrho d^4\varsigma h_{ab}^0(\varrho) h_{cd}^0(\varsigma) \langle \mathcal{T}^{ab}(\varrho) \mathcal{T}^{cd}(\varsigma) \rangle + \mathcal{O}((h^0)^3). \quad (5.4)$$

The potential  $\mathcal{U}$  appearing in the above action was defined in (4.2), and, as before,  $\mathcal{K} = 1 - \sqrt{g^{-1} \gamma_{ab}(\varphi(x))} \partial_\mu \varphi^a \partial_\nu \varphi^b$ . Both the CFT and the massive gravity sectors essentially stay the same (for further comments, see Section 8), despite the global modification. The difference is that the 4D cosmological constant,  $\Lambda$ , is removed from the  $g$  dynamics, and the volume factor  $V_F$  is taken on the bulk solution for  $F$ .

To make things clearer, we emphasize a subtlety in how the effective boundary action (5.4) is obtained. Upon varying  $\mathcal{A}_5$  w.r.t.  $F$  the equations of motion are,  $G_{AB} = (\tilde{\Lambda} - \tilde{S}/2)F_{AB}$ , where  $\tilde{\Lambda}$  is a constant,  $\tilde{S}$  is a constant-valued functional, and  $G_{AB}$  is the Einstein tensor for  $F$ . Thus  $\tilde{S}$  contributes to the bulk CC, and this renormalizes the CC of the  $F$ -universe, as discussed in detail in [5]. We’ve already moved the 4D CC from the  $g$ -universe into the  $F$ -universe, but the  $g$ -universe can also produce a self-accelerated background with curvature  $\sim m^2$ , that would also contribute to the bulk CC.<sup>10</sup> Then, one integrates out  $F$ , as in AdS/CFT, and thus one naively seems to be able to reduce  $\mathcal{A}_5$  to a boundary action dual to a pure, unbroken CFT corresponding to the second line of (5.4). However, this is false: there ought to be an additional contribution to the boundary effective action due to the  $g$ -metric. To see this, we note that variation of  $\mathcal{A}_5$  w.r.t.  $g$  gives rise to 4D equations of motion for dynamical gravity described by the metric  $g$ . In order for the effective boundary action (5.4) to capture this, it has to include the first line. As a result, the  $g$ -metric couples to a fiducial metric,  $\eta + h^0$ , obtained from the bulk dynamics. The fiducial metric at this point is not yet specified and the theory looks like bigravity [19], but with a different action for  $h^0$  given in (5.4) in the lowest order in  $h^0$ . As we’ve already noted at the end of Section 4, the CFT correlators obtained by varying (5.4) w.r.t.  $h^0$ , and subsequently putting  $h^0 = 0$ , will contain local pieces proportional to the graviton mass. Thus CFT correlation functions are amended by local terms proportional to the graviton mass. On the other hand, the resulting gravity equations will be those of massive gravity with Minkowski fiducial metric [8, 9], since  $h^0 = 0$  should be used in the end to comply with the AdS/CFT prescription.

From (5.4), the relevant part of the action is

$$S_g = \frac{V_F}{V_g} M_{\text{Pl}}^2 \int d^4x \sqrt{|g|} [R(g) + 2m^2 \mathcal{U}(\mathcal{K})], \quad (5.5)$$

with the fiducial metric  $\gamma_{\mu\nu} = [\eta_{ab} + h_{ab}^0(\varphi(x))] \partial_\mu \varphi^a \partial_\nu \varphi^b$ , determined by the Dirichlet boundary data  $h_{ab}^0$ , and the volume factor  $V_F$  taken on the bulk solution for  $F$ . Setting  $M_{\text{Pl}}$  to 1,

---

<sup>10</sup>We note however, that the latter is many orders of magnitude smaller compared to a typical curvature in the  $F$ -universe [5], and can be ignored for all “practical purposes” of the bulk physics.

the  $g$ -metric Einstein equations can then be written as nine traceless and one trace equations [5],

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T, \quad (5.6)$$

$$R + T = \langle T \rangle - 2m^2 \left\langle g^{\mu\nu} \frac{\partial \mathcal{U}}{\partial g^{\mu\nu}} \right\rangle, \quad (5.7)$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{|g|}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x \sqrt{|g|} m^2 \mathcal{U} = m^2 \left( g_{\mu\nu} \mathcal{U} - 2 \frac{\partial \mathcal{U}}{\partial g^{\mu\nu}} \right), \quad (5.8)$$

where  $\langle \dots \rangle$  denotes the spacetime average,

$$\langle \dots \rangle \equiv \frac{\int d^4x \sqrt{|g|} (\dots)}{V_g}. \quad (5.9)$$

Note that on self-accelerated solutions,  $R, T \sim m^2$  are equal to constants, thus  $R = \langle R \rangle$  and  $T = \langle T \rangle$  should be used in (5.7). Also,  $g^{\mu\nu} \partial \mathcal{U} / \partial g^{\mu\nu} = C(\alpha_3, \alpha_4)$  is a constant on the self-accelerated solutions, with  $\alpha_3$  and  $\alpha_4$  being the free parameters in (4.2). Its spacetime average yields the same constant, and therefore (5.7) is reduced to

$$R = -2m^2 C(\alpha_3, \alpha_4). \quad (5.10)$$

Hence, for some reasonable choices of parameters, we may get  $m^2 \sim H_0^2$  in a technically natural way, with  $H_0$  being the Hubble constant.<sup>11</sup> This conclusion is not changed by introducing quasidilaton [24], which only affects and improves the dynamics of small perturbations by removing unstable and superluminal modes [25]. Furthermore, instead of massive gravity or quasidilaton, one can straightforwardly use any of their known extensions, see, e.g., [19, 23, 26] – the key point is for the theory to have a fiducial metric and for the dark energy to be given by the stress-energy tensor associated with graviton mass. As such, our approach establishes a theory on the  $\text{AdS}_5$  boundary that removes the big cosmological constant and generates a small curvature given by the graviton mass. Meanwhile, it also ascribes natural meanings to the fiducial metric and the diffeomorphism Stükelberg fields in the massive gravity.

## 6 The Loop Expansion

In this section, we briefly review the quantization algorithm in which the SM fields are quantized with the Planck constant  $\hbar$ , whereas gravity with the rescaled Planck constant,  $\hbar q$ , where  $q$  is a functional whose magnitude is determined by classical equations of motion [5]. In particular, the classical solutions that we consider as relevant for our purposes all give  $q \rightarrow 0$ . Therefore, our theory reduces to one in which all the fields but gravity are

---

<sup>11</sup>Note that in massive gravity one would have gotten a similar equation,  $R = -2m^2 \tilde{C}(\alpha_3, \alpha_4)$ , but with a different function of the parameters,  $\tilde{C}$ ; this is because equation (5.7) differs from the corresponding massive gravity equation,  $R + T = 0$ . How this difference affects the faith of fluctuations on the self-accelerated solution needs to be investigated.

quantized in a conventional manner, while gravity is kept classical, at least at low energies, since  $\hbar q \rightarrow 0$ .

Proposals to quantize the SM, and couple it to classical gravity, have been extensively discussed in the past, see works [27, 28, 29] and references therein. That such a scheme should exist at least as an approximation – to describe our empirical experience with quantized SM and classical gravity so far observed in our universe – is undeniable. More subtle is the question that whether the observed gravity in our universe can be classical as a matter of principle, and be consistently coupled to quantized SM; this is the question addressed affirmatively in [27, 28, 29] and references therein. While in our approach we will not need to assume that a fundamental theory of gravity is not quantized, we will nevertheless consider a case where at low energies the effective Planck’s constant for gravity,  $\hbar q$ , tends to zero, and in that respect, the present section describes a straightforward application of some of the techniques of [27, 28, 29] to our approach. In particular, we will use the method of Ref. [29] (we could use equally well the in-in formalism and its earlier versions reviewed in [28]). While the approach of Ref. [29] corresponds to our case  $\hbar q = 0$ , we argue that the limiting procedure,  $\hbar q \rightarrow 0$ , can also be well defined and meaningful.

We then apply this algorithm to massive gravity, and argue that this procedure simply removes the problem of otherwise unmanageable counterterms at the strong coupling scale of that theory.

## 6.1 Non-gravitational Loops

We assume that classical gravity should be completed into a full-fledged quantum theory with good UV behavior at a certain energy scale,  $M_{\text{QG}}$  (the Planck scale, or string scale), that is higher, by at least an order of magnitude, than the particle physics UV scale,  $M_{\text{SM}}$ . The latter is assumed to be a scale at which the particle physics interactions themselves (with gravity switched off) become UV complete, for instance, in an asymptotically free grand unified theory (GUT). Then, it is not unnatural to expect a hierarchy between  $M_{\text{QG}}$  and  $M_{\text{SM}}$  of two orders of magnitude. For definiteness, we will assume that  $M_{\text{QG}}$  is of order the Planck scale,  $M_{\text{QG}} \sim 10^{18}$  GeV, while  $M_{\text{SM}}$  is of order the GUT scale,  $M_{\text{SM}} \sim 10^{16}$  GeV, but our discussion does not really depend on these concrete values.

Owing to this hierarchy, gravity should be well approximated by a classical field theory at and below the energy scale  $M_{\text{SM}}$ . In this low energy approximation, the path integral can be defined for all the SM fields quantized with  $\hbar$ , while gravity can be regarded as a classical field. For self-consistency, subsequent quantization of gravity should only give rise to negligible corrections to the quantized SM results. Since the corresponding formalism is well known (see [28, 29] and references therein) our discussions in this section will be brief and schematic.

Thus, the Feynman integral for the quantized SM interactions at and below  $M_{\text{SM}}$  reads

$$Z(g, J_n) = \text{const} \times \int d\mu(\tilde{\psi}_n) \exp \left( \frac{i}{\hbar} \int d^4x \sqrt{|g|} \left( \mathcal{L}(g, \tilde{\psi}_n) + J_n \tilde{\psi}_n \right) \right), \quad (6.1)$$

where  $d\mu(\tilde{\psi}_n)$  is the measure for all the SM fields,  $\tilde{\psi}_n$ .<sup>12</sup>

---

<sup>12</sup>Gauge fixing and the Faddeev-Popov determinant are included in this measure.

As we have emphasized, the metric  $g$  is regarded as an external field for now, and so are the sources,  $J_n$ , introduced for every SM field. The 1PI effective action can be defined via a Legendre transform of  $W(g, J_n) = -i \ln Z(g, J_n)$  as follows,

$$\Gamma_{\text{1PI}} \equiv W(g, J_n) - \int d^4x \sqrt{|g|} J_n \psi_n, \quad (6.2)$$

where  $\sqrt{|g|} \psi_n \equiv -i \delta \ln Z(g, J_n) / \delta J_n$  is  $\sqrt{|g|}$  times the expectation value of the SM field  $\tilde{\psi}_n$  in the presence of the source  $J_n$ . These expectation values, upon which the 1PI action depends, are referred to as “classical fields,” to emphasize that in the conventional approach they are not to be quantized further, since all the quantum corrections due to the SM interactions are already taken into account in  $\Gamma_{\text{1PI}}$  and  $\mathcal{L}_{\text{SM}}$ .

The “classical fields”  $\Psi_n$ , as well as the 1PI action,  $\Gamma_{\text{1PI}}$ , are not real in general even for a subset of the original fields,  $\tilde{\Psi}_n$ , that could be real. On the other hand, one does need a real effective action to couple to gravity in a conventional manner in (1.1). As shown in Ref. [29] the real part of the 1PI action,

$$\Gamma_{\text{eff}} \equiv \text{Re } \Gamma_{\text{1PI}} \equiv \int d^4x \sqrt{|g|} \mathcal{L}_{\text{SM}}(g, \psi_n), \quad (6.3)$$

can consistently be constructed and used as a quantum effective action to which classical gravity can couple [29]. Hence, the classical gravity equation would read as follows:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \Gamma_{\text{eff}}. \quad (6.4)$$

What follows in the next subsection is just a straightforward application of this procedure of coupling classical gravity to the quantized SM fields to our case. It amounts to an insertion of the effective action (6.3) into (1.1), to account for dynamical gravity. While for small but nonzero  $q$  this procedure would require more careful study, for  $q \rightarrow 0$  it reduces to the known one [29]; this is enough for our purposes.<sup>13</sup>

## 6.2 Gravity Loops

A quantum theory of gravity is likely to come with new degrees of freedom at the energy scale  $M_{\text{QG}}$ , as does string theory. However, even before the full quantum theory of gravity

---

<sup>13</sup>While we use the technical tool developed in Ref. [29], we also note here a conceptual difference of our framework. In the approach of [29] Einstein’s gravity is considered to be a fundamental classical theory and quantum matter fields are coupled to it. Due to the matter loop corrections, one generates the terms that are quadratic in curvature invariants,  $R_{\mu\nu}^2, \dots$ . No higher powers of curvature get generated since with the quadratic curvature counterterms the theory is shown to be renormalizable [29]. However, it is also not causal since the quadratic curvature invariant terms – when stand alone – generate the Ostrogradsky ghosts, whose removal leads to acausalities. In contrast with this, our approach assumes that gravity is to be completed at some high scale by new physics (such as string theory). Thus there should be an infinite number of curvature terms in the classical action in addition to the Einstein-Hilbert term. These terms do not result from loops, but from a tree-level expansion (say, the  $\alpha'$  expansion in string theory). Moreover, we argue that there are no low energy loop corrections in the gravity sector since  $\hbar q \rightarrow 0$ . Thus, the SM loops will just renormalize already pre-existing  $R^2$  and higher terms, but the series in curvature invariants should not be truncated at any finite order when it comes to the discussion of the Ostrogradsky ghost; hence, neither the existence of such ghosts, nor the acausality can be established in our case.

is explored, there is a more immediate issue within the low energy effective theory: the quantum gravity corrections should be small at momentum/energy scales below  $M_{\text{QG}}$  for a classical treatment to be sensible. For instance, if Einstein's gravity is regarded as part of a low energy quantum effective field theory, the loops only generate higher dimensional operators that make small contributions at momenta/energies below  $M_{\text{QG}}$ . Hence, all the classical predictions of the Einstein theory at length/time scales well above  $M_{\text{QG}}^{-1}$  are intact, to a good approximation. We should then strive to make sure that the quantum corrections are small at large length/time scales in the proposed framework. Given that the classical action has an unusual form in the present case, one first needs to set the rules of calculation for the gravity loops, no matter how small their value could be. These rules were outlined in [5]; we discuss them in more detail here, especially in the context of Ref. [29].

Following [5], we consider the path integral for gravity,

$$Z_g = \text{const} \times \int d\mu(g) d\mu(F) d\lambda dq \exp \left\{ \frac{i}{\hbar} \left( \frac{1}{q} (S_{\text{mGR}} + \Gamma_{\text{eff}}) + \lambda((V_g/V_F) - q) + S_F \right) \right\}, \quad (6.5)$$

where  $S_{\text{mGR}}$  is the massive gravity action for the  $g$ -universe, the two measures  $d\mu(g)$  and  $d\mu(F)$  include the gauge fixing conditions and the Faddeev-Popov determinants for both  $g$  and  $F$ , and  $\Gamma_{\text{eff}}$  is the real part of the 1PI effective action for the SM fields discussed in the previous subsection.  $\Gamma_{\text{eff}}$  contains real parts of all possible Green's functions of the SM fields, with gravity treated as an external field. The imaginary parts for these Green's functions can be restored from the real parts of the lower order Green's functions, by using the optical theorem [29]. Thus,  $\Gamma_{\text{eff}}$  contains in principle all the information about the quantized SM in classical gravitational field.

The above path integral defines an algorithm, albeit unconventional and a bit cumbersome, for calculating quantum loops: to reiterate, the SM loops are done in a conventional way using  $\hbar$  while treating  $g$  and  $F$  as external classical fields; this gives rise to  $\Gamma_{\text{eff}}$  defined in the previous subsection. Then, the gravity loops are done using (6.5); these loops would contain only real parts of the SM Green's functions since  $\Gamma_{\text{eff}} = \text{Re } \Gamma_{1\text{PI}}$ . The respective imaginary parts can be obtained from the optical theorem for the SM loops; in our case they should be added to the real parts of the SM Green's functions that would appear in the order-by-order expansion in (6.5) (one can of course get both the real and imaginary parts of those Green's functions if one calculates  $\Gamma_{1\text{PI}}$ ; it's also well known that the solutions obtained from  $\Gamma_{1\text{PI}}$  are gauge independent).

In this scheme, the parameter  $\hbar q$  may be regarded as a second Planck constant that governs the gravity loops at low energies. Since the classical solutions in our case are such that  $\hbar q \rightarrow 0$ , and hence no gravity loop contributions survive, it is then self-consistent to use classical gravity equations (6.4).<sup>14</sup>

---

<sup>14</sup>The obtained action,  $S_{\text{gravity}} + \Gamma_{\text{eff}}$ , can be used to develop a tree-level nonlinear perturbation theory to account for all the known classical gravity, astrophysics, and cosmology effects. One can try to go further and address a more ambitious program of defining the theory for  $q \neq 0, q \neq 1$ . This is not our goal here, but we make a few comments on such a prospect: In that case, the mixed gravity-SM quantum loops can be done by integrating in (6.5) further with respect to the SM "classical fields" upon which  $\Gamma_{\text{eff}}$  depends. This integration leads to a "double" quantization of the SM fields – the primary quantization of the SM fields being done with  $\hbar$ , and the secondary quantization of the "classical" SM fields appearing in the effective action, being done with the rescaled constant  $\hbar q$ . As long as  $\hbar q \rightarrow 0$ , the secondary quantization does

With the quantization process defined as above, one needs to worry about two types of quantum corrections that may potentially ruin the classical solution of the cosmological constant problem in the  $g$ -universe: (A) The action (1.1) and (2.1) appear to suggest that there should be novel nonlocal interactions in the  $g$ -universe, arising due to the product of two integrals in  $V_g^{-1}S_{\text{mGR}}$ ; (B) The form of the effective action in (2.2) and (6.5) raises the question of whether new polynomial terms of  $\lambda$  and  $q$  may be generated by quantum corrections, and whether these new terms can spoil the classical solutions.

To address the point (A), let us decompose the metric as a background,  $g_b$ , and its fluctuation, schematically  $g = g_b + h$ . The inverse volume factor,  $V_g^{-1}$ , multiplying the action  $S_{\text{mGR}}$ , can then be expanded as follows:  $V_g^{-1} = V_b^{-1} - V_b^{-2}H_h + \dots$ , where  $V_b = \int d^4x \sqrt{|g_b|}$  and  $H_h = \int d^4x \sqrt{|g_b|} h/2$ . The term,  $-V_b^{-2}H_h S_{\text{mGR}}$ , as well as the other terms containing higher powers of  $h$ , will produce new unconventional interaction vertices at both the tree level and in the loops. When sandwiched between various states, the tree-level terms either give trivially zero, or are suppressed by extra powers of the inverse volume,  $V_b^{-1}$  as long as all the fields involved decay at spatial/time infinity; they can also give rise to amplitudes that do not correspond to any scattering process.<sup>15</sup> As to the loops arising from these novel vertices, they will be suppressed by powers of  $V_b^{-1}$ , in addition to being governed by the vanishing effective Planck constant,  $\hbar q$ .

To address the point (B), let us rewrite the partition function in a slightly different form:

$$\int d\mu(g) d\mu(F) d\lambda dp \exp \left\{ \frac{i}{\hbar} \left( \frac{V_F}{p} (S_{\text{mGR}} + \Gamma_{\text{eff}}) + \lambda(V_g - p) + S_F \right) \right\}, \quad (6.6)$$

where we integrate with respect to  $p$  and  $\lambda$ . Since there is no Wick contraction between  $g$  and  $F$ , the multiplier  $V_F$  in front of  $S_{\text{mGR}} + \Gamma_{\text{eff}}$  does not produce any new vertices in the  $g$ -universe. Hence, this multiplier can be regarded as a pure number from the point of view of the  $g$ -universe. The new nonlocal vertices described in question (A) are now encoded in the conventional-looking term  $\lambda V_g$ . These new vertices will modify the calculations, but the modifications are straightforward to take into account. To do so, let us further rewrite the

---

not modify the results of the primary quantization of the SM fields. However, the procedure may lead to unconventional results if  $q \neq 0, q \neq 1$ , and this needs to be studied further. All we can say here is that in the limit  $\hbar q \rightarrow 0$  the optical theorem is valid even for gravity and mixed loops order by order in  $\hbar q$ , given that one incorporates the optical theorem for the SM Green's functions to restore their respective imaginary parts as discussed above; the optical theorem for the gravity and mixed loops would relate tree amplitudes to the respective loop amplitudes, even though the loop-generated counterterms to the effective action tend to zero as  $\hbar q \rightarrow 0$ . For instance, consider a one-loop diagram that describes a self-energy correction due to a graviton to one of the "classical" SM fields appearing in  $\Gamma_{\text{eff}}$ . This diagram has two vertices, each proportional to  $1/\hbar q$ , and two propagators, each proportional to  $\hbar q$ , hence the diagram is  $\mathcal{O}(1)$ , if we choose to normalize the one-particle states so that the external lines in the diagram carry no powers of  $\hbar q$ . The imaginary part of this diagram, which is also  $\mathcal{O}(1)$ , describes a classical process of emission of a graviton by a SM particle. This amplitude is certainly nonzero in the  $\hbar q \rightarrow 0$  limit. However, the loop diagram itself does not introduce any counterterms in the partition function – being  $\mathcal{O}(1)$ , it is suppressed by  $\hbar q$ , as compared to the leading tree-level terms in the path integral that are of the order  $1/\hbar q$ . We will not pursue here these arguments further, and will restrict to the limit  $\hbar q \rightarrow 0$  for the remainder of the paper. We thank V.P. Nair for useful discussions of the above issues.

<sup>15</sup>Similar amplitudes are also present in ordinary theories at higher orders in  $1/\hbar$ . For instance, in a theory of a massive scalar  $\phi$ , the tree amplitude obtained by sandwiching the operator  $\hbar^{-2} \int d^4x m^2 \phi^2(x) \times \int d^4y m^2 \phi^2(y)$  between two two-particle states does not correspond to any scattering process.



partition function,

$$\int d\mu(g)d\mu(F)d\lambda dp \exp \left\{ \frac{i}{\tilde{\hbar}} \left( S_{\text{mGR}} + \Gamma_{\text{eff}} + \tilde{\lambda}(V_g - p) \right) + \frac{i}{\tilde{\hbar}} S_F \right\}, \quad (6.7)$$

where  $\tilde{\hbar} = \hbar p/V_F$ , and  $\tilde{\lambda} = \lambda p/V_F$ . Then, we consider loop corrections about a flat  $g$ -background, which is a solution for arbitrary CC in the  $g$ -universe. Due to these corrections there will be additional terms proportional to positive powers of  $\tilde{\hbar}$ , which need to be included in the effective action for gravity. The most dangerous of these terms would be proportional to

$$\tilde{\hbar}^{k+1} \tilde{\lambda}^{l+2}, \quad k, l = 0, 1, 2, 3, \dots \quad (6.8)$$

However, it is straightforward to see that on the classical solutions of the original theory, all these terms vanish when  $V_g/V_F \rightarrow 0$ .

We end this section by commenting on another type of corrections that are likely to exist in the theory. The action  $S_{\text{mGR}}$  contains the Einstein-Hilbert term, but it may also contain higher derivative terms, such as higher powers of curvature invariants, that are not necessarily induced by quantum loops. These terms can arise in a putative UV completion of gravity. In string theory, for instance, these terms would be due to the  $\alpha'$  corrections. Such terms will not spoil any of our arguments as long as we are considering energies and momenta below the scale by which these higher dimensional operators are suppressed in comparison with the EH term. Since the latter scale should be expected to be of order  $M_{\text{QG}}$ , the effects of these higher dimensional operators are then negligible for the SM fields at and below  $M_{\text{SM}}$ . On the other hand, the higher dimensional operators will be relevant for physics at very short distances. One can hope that these operators will smooth out the short length/time singularities of certain classical solutions, e.g., black holes, or cosmological solutions. Similar considerations apply to higher dimensional operators that are also expected to appear in  $S_F$ .

## 7 On Strong Coupling in Massive Gravity

The diffeomorphism invariant action for massive gravity was built in [8, 9], and was presented in (4.1). The specific structure of the potential guarantees that the theory propagates only 5 degrees of freedom [8, 9, 18, 30, 31, 32, 33]. It is not guaranteed, however, that the above structure is preserved by loop corrections. We note that the coefficients  $\alpha_{3,4}$  in (4.1) get renormalized only multiplicatively [13], i.e. if set to zero they remain zero, but the loops would in general induce other terms such as  $\det_3(\mathcal{K})^2$  or  $\det_3(\mathcal{K}) \times \det_4(\mathcal{K})$ , etc., that would reintroduce the sixth ghostly degree of freedom at a certain energy scale.

To see this more explicitly, we take the so-called decoupling limit  $M_{\text{Pl}} \rightarrow \infty$ ,  $m \rightarrow 0$ , with  $\Lambda_3 = (M_{\text{Pl}} m^2)^{1/3}$  fixed [34]. In this limit, the five polarizations of the massive graviton acquire their individual identities as the helicity  $\pm 2$ , helicity  $\pm 1$  and helicity 0 states. For simplicity, we focus on the helicity-0 state, which we denote by  $\pi$ . The massive gravity action in the decoupling limit contains the following terms for the  $\pi$  field:

$$S_\pi = \int d^4x \left( -\frac{1}{2}(\partial\pi)^2 + \alpha \frac{(\partial\pi)^2 \square\pi}{\Lambda_3^3} + \frac{2\alpha^2}{3} \frac{(\partial\pi)^2 ((\partial\partial\pi)^2 - (\square\pi)^2)}{\Lambda_3^6} + \dots \right). \quad (7.1)$$

These are the so-called Galileon terms [35]; they are special since they don't generate higher than two derivatives in the equation of motion, thus retaining only one degree of freedom in the  $\pi$  theory on an arbitrary background.

On the other hand, it is evident from the above action that interactions of the  $\pi$  field are described by irrelevant operators, that become strong at the scale  $\Lambda_3$ . If this theory is quantized in a conventional way, the renormalized Lagrangian would include an infinite number of counterterms with higher derivatives, such as higher powers of  $\partial\partial\pi$ .

These new terms, if present, would introduce a ghost at the scale  $\Lambda_3$ . One way to deal with this problem is to regard the action (7.1) as an effective action valid below the scale  $\Lambda_3$ , above which it needs to be completed by some unknown new physics. The effects of the new physics in the low energy theory would manifest themselves as an infinite series of higher derivative operators suppressed by  $\Lambda_3$ . The appearance of the ghost at that scale could then be attributed to the artificial truncation of the series at a finite order. Such an approach is possible, and has so far been often adopted. It calls for an answer to the question as to what is the completion of massive gravity at the scale  $\Lambda_3$ , especially given that the latter is so much smaller than  $M_{\text{Pl}}$  (see [10, 11], and references therein).

In our approach, however, such a question does not arise. This is because the massive gravity Lagrangian is not quantized with  $\hbar$ , but instead with  $\hbar q$ . The appropriate part of the path integral reads schematically as

$$\exp\left(\frac{i}{\hbar q}S_\pi\right). \quad (7.2)$$

Thus, every  $\pi$  propagator will be proportional to  $\hbar q$  and every  $\pi$  vertex, be it cubic or quartic, to  $1/\hbar q$ . If we normalize one-particle states in a way that the standard loop expansion is an expansion in powers of the respective Planck constant, then all the loop-generated counterterms will be proportional to positive powers of  $\hbar q$ , and would vanish in the setup considered here.

We note that this does not obviate the strong coupling scale  $\Lambda_3$ , but only renders the full quantum theory with a finite number of tree-level terms, so the theory is still strongly coupled. While there are no quantum mechanical constraints imposed on the amplitudes, since  $\hbar q \rightarrow 0$ , a resummation of the classical nonlinear diagrams would be needed to account for the Vainshtein effect [36] (see also [37]). It is also relevant to note that the tree amplitudes define loop level amplitudes via the optical theorem which remains valid order-by-order, but the effective action receives no loop-generated counterterms. This statement is trivial if we restrict ourselves to the  $\pi$ -sector only – that sector has only one effective Planck's constant,  $\hbar q$ , hence the reason for validity of the optical theorem is identical to that in a conventional theory.<sup>16</sup> To see how this works in more detail, consider the  $2 \rightarrow 2$  scattering amplitude at the one-loop level that is due to the Wick contraction of two quartic Galileon terms. We recall that every propagator will be proportional to  $\hbar q$ , and every tree level vertex will be proportional to  $1/\hbar q$ . Moreover, if we normalize one-particle states in a way that the external lines in Feynman diagrams carry no factors of  $\hbar q$ , then, the one loop diagram for the  $2 \rightarrow 2$  scattering amplitude that comes from a contraction of two quartic Galileons would be  $\mathcal{O}(1)$ .

---

<sup>16</sup>However, the situation changes when one couples the  $\pi$ -sector with the SM fields that are now quantized with a different Planck's constant,  $\hbar q$ . The validity of the optical theorem for  $q \neq 0, q \neq 1$  is plausible, but has not been demonstrated, as discussed in Footnote 14.

The imaginary part of this diagram is also of order  $\mathcal{O}(1)$ , and this is equated to a square of the tree-level diagram, times two powers of  $\hbar q$  coming from the insertion of the intermediate states, ensuring the validity of the optical theorem. While the imaginary part of this one-loop diagram would satisfy the optical theorem, the diagram itself would not give rise to a counterterm in the exponent of the path integral, since this counterterm would be of order  $\mathcal{O}(1)$ , while the existing tree-level terms in the exponent are of order  $1/\hbar q$ . As to the  $2 \rightarrow 2$  amplitude itself, it also receives a dominant tree-level contribution from the contraction of two cubic Galileon terms, and the corresponding amplitude – with the normalization chosen here for the external state – is of order,  $\hbar q \times (1/\hbar q)^2 = 1/\hbar q$ .

Thus, the full quantum effective action for the  $\pi$  field has as few terms as the classical action. All the calculations done in this theory – although nonperturbative – are exact. In other words, all the Feynman diagrams are defined just by the tree-level action. For external classical sources, the Vainshtein mechanism [36] will postpone the strongly coupled regime to energy/momentum scales higher than  $\Lambda_3$  (see [38] for a review) .

## 8 Comments and Outlook

This section consists of a few comments on topics that are somewhat disconnected from each other, but might be helpful to further explore the present proposal.

First we note that since  $\hbar q \rightarrow 0$ , low energy gravity is essentially classical, and therefore, there will be no tensor modes generated by quantum fluctuations during inflation. The scalar mode will still be generated since the inflaton is quantized in a conventional way, and even though the scalar fluctuation is a mixture of inflaton and metric fluctuations, one could choose a gauge in which the scalar perturbation is due entirely to an inflaton<sup>17</sup>; thus, our framework would not change the inflationary predictions for the scalar perturbations. However, if the primordial quantum tensor modes are discovered this would rule out our proposal with  $\hbar q \rightarrow 0$ .

In the context of the present work, we argued that it could be advantageous to consider massive gravity as an agent driving the accelerated expansion of the universe. However, as we have mentioned, massive gravity is not the only way. One could easily imagine a quintessence field protected by symmetries, as in the pseudo-Nambu-Goldstone boson (PNGB) scenario of [39], to give rise to dark energy. It is straightforward to see that all our requirements would remain intact if we assumed that the PNGB gave rise to dark energy in the  $g$ -universe on the boundary of  $\text{AdS}_5$ . While the  $g$ -universe in that case would tend to 4D Minkowski space in the infinite future, the ratio  $V_g/V_F$  would still tend to zero since  $V_F$  (= volume of  $\text{AdS}_5$ ) is infinitely larger than the volume of infinite 4D Minkowski space. While the PNGB model would give a redshift dependent equation of state for dark energy, massive gravity and its extensions give the redshift-independent equation,  $p = -\rho$ .

It is worth noting that the action (5.1) allows for straightforward Euclidean extensions, since the standard rotation to Euclidean times,  $t_E = it$ , done in both universes simultaneously, amounts to the usual Euclidean transition in the generating functional. Thus, the solutions that have real Euclidean counterparts (say, maximally symmetric spaces) can be

---

<sup>17</sup>One could see this also without choosing a gauge but by diagonalizing and rescaling the system of inflaton and metric perturbations in the scalar sector.

discussed using the Euclidean path integral. One could study in more detail the AdS/CFT algorithm in Euclidean space, and see how it is affected when the  $V_F/V_g$  factor is introduced.<sup>18</sup>

We'd like to comment on the RS2 model [40], where induced 4D gravity exists on the RS brane (set  $z = \epsilon > 0$  and impose  $Z_2$  across the hypersurface in Sections 4 and 5). This scenario would allow the boundary metric  $\gamma_{ab}$  to be truly dynamical. For instance, its linearized Einstein kinetic term is given in (4.17), from which it is clear that a mass scale proportional to  $1/\epsilon$  acts as the effective Planck mass for  $\gamma_{ab}$ . Then, the braneworld will contain a ghost-free bi-gravity theory in the framework of Section 4 [19], or a globally connected bi-metric theory in that of Section 5. However, the value of the CC would need to be fine-tuned in order to fulfill a set of junction conditions that maintain the consistency of the RS2 picture, in which case our solution to the big CC problem would be lost. In retrospect, the fine-tuning that precludes the proposed solution of the CC problem to be applicable in the RS2 scenario, is not required on the conformal boundary of AdS<sub>5</sub> (i.e., when  $\epsilon \rightarrow 0$ ) because there is no  $F$ -metric dynamics in the  $\epsilon \rightarrow 0$  limit.

Last but not least, it would be interesting to look into the vierbein formulation of massive gravity [41, 42] in this scenario: besides the Stückelberg fields  $\varphi^a$ , the vierbein formulation requires a two-index field  $\lambda_a^\alpha$  [42]. It would be fascinating if one were able to understand the origin of this field from a broader context.

## Acknowledgements

We are grateful to A. Kobakhidze and V.P. Nair for useful discussions. Both GG and SY are supported by NASA grant NNX12AF86G S06, and GG also by NSF grant PHY-1316452. GG acknowledges a membership at the NYU-ECNU Joint Physics Research Institute in Shanghai.

## A Boundary Fields in AdS/CFT

In this section, we switch to the Euclidean signature, in tandem with the standard language of AdS/CFT. We present how to find the dual boundary CFT action for bulk graviton. As a precursor, we discuss the massless scalar case first, which is simpler but very similar to the graviton case.

### A.1 Massless Scalar

The Euclidean AdS<sub>5</sub> metric is

$$ds^2 = F_{AB} dx^A dx^B = \frac{l^2}{z^2} (dz^2 + d\vec{x}^2). \quad (\text{A.1})$$

---

<sup>18</sup>Without this factor, the only novelty in our case is that the full boundary action would become  $S_{\text{mGR}}[g, \gamma] + S_{\text{CFT}}$ .

We set  $l = 1$  in the following discussion and only restore it when necessary. The scalar action is

$$S = \frac{1}{2} \int d^4x dz \sqrt{F} F^{AB} \partial_A \Phi \partial_B \Phi, \quad (\text{A.2})$$

whose variation gives the equation of motion

$$\left( \partial_z^2 - \frac{3}{z} \partial_z + \square \right) \Phi(z, x) = 0. \quad (\text{A.3})$$

The general solution that decays at the horizon  $z = \infty$  is  $\Phi \sim z^2 K_2(z\sqrt{-\square}) \phi_0(x)$ . Integrating by parts and using equation of motion,

$$S = \frac{1}{2} \int d^4x \sqrt{\tilde{\gamma}} \Phi \hat{n} \cdot \nabla \Phi, \quad (\text{A.4})$$

where  $\tilde{\gamma}_{ij}$  is the induced metric on the boundary and  $\hat{n} \cdot \nabla = -z \frac{\partial}{\partial z}$ . To obtain the effective boundary action, we set  $z = \epsilon$  and take  $\epsilon \rightarrow 0$  in the end. We call the boundary condition  $\Phi(\epsilon, x) = \phi_0(x)$ , so that

$$\Phi(z, p) = \frac{z^2 K_2(pz)}{\epsilon^2 K_2(p\epsilon)} \phi_0(p) \quad (\text{A.5})$$

in the Fourier space.

Plugging this into (A.4),

$$S = -\frac{1}{2} \epsilon^{-4} \int d^4x d^4y \phi_0(x) \phi_0(y) \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} u \frac{\partial}{\partial u} \ln(u^2 K_2(u)), \quad (\text{A.6})$$

where  $u = p\epsilon$ . We then expand the integrand in series,

$$u \frac{\partial}{\partial u} \ln(u^2 K_2(u)) = -\frac{u^2}{2} + \frac{u^4}{4} (\ln 2 - \gamma - \ln u) + \dots \quad (\text{A.7})$$

Here,  $\gamma = 0.5772\dots$  is the Euler-Mascheroni constant. The higher order terms in the series can be ignored because they contain more than four powers of  $\epsilon$  and drop out when  $\epsilon \rightarrow 0$ .

After performing the Fourier transform, we have<sup>19</sup>

$$\begin{aligned} S[\phi_0] = & -\frac{\epsilon^{-2}}{4} \int d^4x \phi_0(x) \square \phi_0(x) \\ & + \frac{1}{32\pi^2} \int d^4x d^4y \phi_0(x) \phi_0(y) \square_x^3 \left[ \frac{\ln(|x-y|/\epsilon)}{|x-y|} \right]. \end{aligned} \quad (\text{A.8})$$

The content of the second line can be clarified by introducing an arbitrary and finite mass scale  $\mu$ ,

$$\begin{aligned} \square_x^3 \left[ \frac{\ln(|x-y|/\epsilon)}{|x-y|} \right] &= \frac{1}{2} \square_x^3 \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right] - \square_x^3 \left[ \frac{\ln(\mu\epsilon)}{|x-y|^2} \right] \\ &= \frac{1}{2} \square_x^3 \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right] + 4\pi^2 \ln(\mu\epsilon) \square_x^2 \delta(x-y). \end{aligned} \quad (\text{A.9})$$

---

<sup>19</sup>The relevant formulae are given in [43, 44], where the nonlocal expressions are “differentially regularized” to give well-defined Fourier transform.

The first term is the “differentially regularized” two-point correlator for the boundary scalar CFT operator, up to a constant multiple:<sup>20</sup>

$$\frac{\pi^2}{24} \langle \mathcal{O}(x) \mathcal{O}(y) \rangle_{\text{reg}} = -\frac{1}{2^7 \cdot 6} \square_x^3 \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right] \quad (\text{A.10})$$

$$= \lim_{\kappa \rightarrow 0} \left\{ \frac{1}{|x-y|^{8+2\kappa}} + \frac{\mu^{2\kappa}}{\kappa} \frac{\pi^2}{2^5 \cdot 6} \square_x^2 \delta(x-y) \right\}, \quad (\text{A.11})$$

so the well-known  $\sim |x-y|^{-8}$  two-point function is recovered [45]. The total boundary action is

$$S[\phi_0] = -\frac{1}{2} \int d^4x \phi_0(x) \left( \frac{\epsilon^{-2}}{2} \square - \frac{\ln(\mu\epsilon)}{4} \square^2 \right) \phi_0(x) - \frac{1}{2} \int d^4x d^4y \phi_0(x) \phi_0(y) \langle \mathcal{O}(x) \mathcal{O}(y) \rangle_{\text{reg}}. \quad (\text{A.12})$$

In the AdS/CFT context, the local terms that diverge in the  $\epsilon \rightarrow 0$  limit are unphysical and should be removed through holographic renormalization [46]. Since the regularized correlator only differs from the unregularized one by local terms, specific renormalization schemes can be constructed to yield either form of the correlator. Still, we note that the differentially regularized expression of the two-point function emerges automatically in the calculation.

## A.2 Graviton

Now we can look at the boundary theory induced by a free graviton from the AdS<sub>5</sub> bulk. The complete action is [22, 47]

$$S = - \int_M d^4x dz \sqrt{F}(R+12) - 2 \int_{\partial M} d^4x \sqrt{\tilde{\gamma}} K + 6 \int_{\partial M} d^4x \sqrt{\tilde{\gamma}}, \quad (\text{A.13})$$

up to a functional of  $\tilde{\gamma}_{ij}$ . We set the Planck scale  $M_5$  to unity for convenience, and use  $i, j, \dots = 1, 2, 3, 4$  to denote the coordinates transverse to  $z$ , which are raised by  $\tilde{\gamma}^{ij}$  on the boundary.

The graviton is defined perturbatively by  $F_{AB} = F_{AB}^{(0)} + h_{AB}$  with background solution  $F_{AB}^{(0)}$ . We work in the gauge  $h_{0B} = 0$ . In the equations of motion, we use  $h_j^i = \tilde{\gamma}^{ik} h_{kj}$  as the dynamical field, since it is related to the boundary condition in a simple way,  $h_j^i|_{z=\epsilon} \equiv \chi_j^i = h_{ab}^0$ , where  $h_{ab}^0$  is the Dirichlet boundary data.<sup>21</sup> This allows the latin indices to be raised and contracted with  $\delta^{ij}$  on the boundary, which we will do from now on. The equations of motion are [47, 48]

$$\partial_z^2 h_j^i + \square h_j^i - \frac{3}{z} \partial_z h_j^i - \frac{1}{z} \partial_z h \delta_j^i + \partial^i \partial_j h - \partial^i \partial_l h_j^l - \partial_j \partial^l h_l^i = 0, \quad (\text{A.14})$$

with constraints

$$\square h - \partial_i \partial^i h_j^j - \frac{3}{z} \partial_z h = 0, \quad (\text{A.15})$$

<sup>20</sup>See Appendix B of [43]. Our formulae are obtained by setting  $k = 2$  there.

<sup>21</sup>Here,  $a, b, \dots = 1, 2, 3, 4$  are raised with  $\delta^{ab}$ . In the case that the boundary condition is related to the fiducial metric of massive gravity, the fiducial metric is  $f_{\mu\nu} = (\eta_{ab} + h_{ab}^0) \partial_\mu \varphi^a \partial_\nu \varphi^b$ .

$$\partial_z(\partial_i h - \partial_j h_i^j) = 0. \quad (\text{A.16})$$

(A.14) implies that the transverse-traceless component of  $h_j^i$  satisfies the free massless scalar equation of motion. We can then write the solution of  $h_j^i$  in Fourier components [47],

$$h_j^i(z, x) = \int \frac{d^4 p}{(2\pi)^4} e^{ipx} \left[ \bar{h}_j^i \frac{z^2 K_2(pz)}{\epsilon^2 K_2(p\epsilon)} - \frac{z^2 - \epsilon^2}{6} \left( p^i p_j \hat{h} - \frac{p^i p_j p^k p_l}{p^2} \hat{h}_k^l \right) \right. \\ \left. + \frac{p_j p^l}{p^2} \hat{h}_l^i + \frac{p^i p_l}{p^2} \hat{h}_j^l - \frac{p^i p_j p^k p_l}{p^4} \hat{h}_k^l + \frac{1}{3} \left( \delta_j^i - \frac{p^i p_j}{p^2} \right) \left( \hat{h} - \frac{p^k p_l}{p^2} \hat{h}_k^l \right) \right], \quad (\text{A.17})$$

where  $\hat{h}_j^i(p)$  is the Fourier transform of  $h_j^i|_{z=\epsilon} = \chi_j^i$ , and

$$\bar{h}_j^i = \hat{h}_j^i - \frac{p_j p^l}{p^2} \hat{h}_l^i - \frac{p^i p_l}{p^2} \hat{h}_j^l + \frac{p^i p_j p^k p_l}{p^4} \hat{h}_k^l - \frac{1}{3} \left( \delta_j^i - \frac{p^i p_j}{p^2} \right) \left( \hat{h} - \frac{p^k p_l}{p^2} \hat{h}_k^l \right). \quad (\text{A.18})$$

Using equation of motion and integrating by parts, the action becomes [22, 47, 48]

$$S = -\frac{1}{4} \epsilon^{-3} \int d^4 x (h_i^j \partial_z h_j^i - h \partial_z h)|_{z=\epsilon}. \quad (\text{A.19})$$

We can substitute (A.17) and (A.18) into this expression to obtain an effective action for  $\chi_j^i$ . The nonlocal terms from this action are worked out in [22, 47]. Here, we give the local terms and the differentially regularized two-point function. The total effective action is

$$S[\chi] = S_{\partial^2} + S_{\partial^4} + S_{\text{nonlocal}}, \quad (\text{A.20})$$

analogous to the massless scalar case. To obtain the results below, 4D integration by parts is used when necessary.

$S_{\partial^2}$  receives contribution from both terms in (A.19):

$$S_{\partial^2} = \frac{\epsilon^{-2}}{2} \int d^4 x \left[ -\frac{1}{4} \chi_i^j \square \chi_j^i + \frac{1}{2} \chi_i^j \partial_j \partial^l \chi_l^i + \frac{1}{4} \chi \square \chi - \frac{1}{2} \chi_i^j \partial^i \partial_j \chi \right], \quad (\text{A.21})$$

which has the structure of the kinetic terms in linearized Einstein gravity, as expected.

Both  $S_{\partial^4}$  and  $S_{\text{nonlocal}}$  come from the first term in (A.19) only. Like the massless scalar case, the arbitrary mass scale  $\mu$  is introduced to the separate these terms. In particular,

$$S_{\partial^4} = \frac{\ln(\mu\epsilon)}{16} \int d^4 x \chi_i^j D_{jl}^{ik} \chi_k^l, \quad (\text{A.22})$$

where the fully symmetrized differential operator  $D_{jl}^{ik}$  is

$$D_{jl}^{ik} = \frac{1}{2} \left( \delta_l^i \delta_j^k + \delta^{ik} \delta_{jl} - \frac{2}{3} \delta_j^i \delta_l^k \right) \square^2 + \frac{2}{3} \partial^i \partial_j \partial^k \partial_l \\ - \frac{1}{2} \left( \delta_l^i \partial_j \partial^k + \delta^{ik} \partial_j \partial_l + \delta_j^k \partial^i \partial_l + \delta_{jl} \partial^i \partial^k - \frac{2}{3} \delta_l^k \partial^i \partial_j - \frac{2}{3} \delta_j^i \partial^k \partial_l \right) \square. \quad (\text{A.23})$$

Also,

$$S_{\text{nonlocal}} = \frac{1}{128\pi^2} \int d^4 x d^4 y \chi_i^j(x) \chi_k^l(y) \square_x D_{jl}^{ik}(x) \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right]. \quad (\text{A.24})$$

We can extract from this the regularized two-point function for the graviton CFT operator,

$$\langle \mathcal{T}_j^i(x) \mathcal{T}_l^k(y) \rangle_{\text{reg}} = -\frac{1}{64\pi^2} \square_x D_{jl}^{ik}(x) \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right]. \quad (\text{A.25})$$

We must show that this is consistent with the well-known unregularized expression [22, 47],

$$\begin{aligned} \langle \mathcal{T}_j^i(x) \mathcal{T}_l^k(y) \rangle &= \frac{20}{\pi^2} \frac{1}{|x-y|^8} \\ &\times \left[ \frac{1}{2} J_l^i(x-y) J_j^k(x-y) + \frac{1}{2} J^{ik}(x-y) J_{jl}(x-y) - \frac{1}{4} \delta_j^i \delta_l^k \right], \end{aligned} \quad (\text{A.26})$$

where  $J_j^i(x) = \delta_j^i - 2 \frac{x^i x_j}{x^2}$ . It is straightforward to do so. From Appendix B of [43] (also Appendix A of [44]), we have the identities

$$\lim_{\kappa \rightarrow 0} \left\{ \frac{1}{|x-y|^{4+2\kappa}} + \pi^2 \frac{\mu^{2\kappa}}{\kappa} \delta(x-y) \right\} = -\frac{1}{4} \square_x \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right], \quad (\text{A.27})$$

$$\lim_{\kappa \rightarrow 0} \left\{ \frac{1}{|x-y|^{6+2\kappa}} + \frac{\pi^2}{8} \frac{\mu^{2\kappa}}{\kappa} \square_x \delta(x-y) \right\} = -\frac{1}{32} \square_x^2 \left[ \frac{\ln(|x-y|^2 \mu^2)}{|x-y|^2} \right]. \quad (\text{A.28})$$

Plugging the non-divergent nonlocal parts of these and (A.10) (terms with negative powers of  $|x-y|$ ) into (A.25) gives (A.26) after some work. We note that this is a more rigorous calculation than the result in [47], which arrives at the same expression but does not address the subtlety of regularizing Fourier transform near poles.

## References

- [1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
- [2] J. Polchinski, hep-th/0603249.
- [3] A. A. Tseytlin, Phys. Rev. Lett. **66**, 545 (1991).
- [4] A. D. Linde, Phys. Lett. B **200**, 272 (1988).
- [5] G. Gabadadze, Phys. Lett. B **739**, 263 (2014) [arXiv:1406.6701 [hep-th]].
- [6] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and G. Gabadadze, hep-th/0209227.
- [7] A. Davidson and S. Rubin, Class. Quant. Grav. **26**, 235019 (2009) [arXiv:0905.0661 [gr-qc]].
- [8] C. de Rham and G. Gabadadze, Phys. Rev. D **82**, 044020 (2010) [arXiv:1007.0443 [hep-th]].
- [9] C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011) [arXiv:1011.1232 [hep-th]].



- [10] K. Hinterbichler, *Rev. Mod. Phys.* **84**, 671 (2012) [arXiv:1105.3735 [hep-th]].
- [11] C. de Rham, *Living Rev. Rel.* **17**, 7 (2014) [arXiv:1401.4173 [hep-th]].
- [12] A. S. Goldhaber and M. M. Nieto, *Rev. Mod. Phys.* **82**, 939 (2010) [arXiv:0809.1003 [hep-ph]].
- [13] C. de Rham, G. Gabadadze, L. Heisenberg and D. Pirtskhalava, *Phys. Rev. D* **87**, no. 8, 085017 (2013) [arXiv:1212.4128].
- [14] G. Gabadadze, K. Hinterbichler, D. Pirtskhalava and Y. Shang, *Phys. Rev. D* **88**, no. 8, 084003 (2013) [arXiv:1307.2245].
- [15] N. Rosen, *Phys. Rev.* **57**, 147 (1940).  
J. Wess and B. Zumino, in B. Zumino, “Effective Lagrangians and Broken Symmetries”, in Brandeis Univ. Lectures on Elementary Particles and Quantum Field Theory (MIT Press Cambridge, Mass.), Vol. 2, 1970, 437.
- [16] C. J. Isham, A. Salam and J. A. Strathdee, *Phys. Rev. D* **3**, 867 (1971).  
A. Salam and J. A. Strathdee, *Phys. Rev. D* **16**, 2668 (1977).
- [17] D. G. Boulware and S. Deser, *Phys. Rev. D* **6**, 3368 (1972).
- [18] S. F. Hassan and R. A. Rosen, *Phys. Rev. Lett.* **108**, 041101 (2012) [arXiv:1106.3344 [hep-th]].
- [19] S. F. Hassan and R. A. Rosen, *JHEP* **1202**, 126 (2012) [arXiv:1109.3515 [hep-th]].
- [20] K. Hinterbichler and R. A. Rosen, *JHEP* **1207**, 047 (2012) [arXiv:1203.5783 [hep-th]].
- [21] C. Deffayet, J. Mourad and G. Zahariade, *JCAP* **1301**, 032 (2013) [arXiv:1207.6338 [hep-th]].  
C. Deffayet, J. Mourad and G. Zahariade, *JHEP* **1303**, 086 (2013) [arXiv:1208.4493 [gr-qc]].
- [22] H. Liu and A. A. Tseytlin, *Nucl. Phys. B* **533**, 88 (1998) [hep-th/9804083].
- [23] S. F. Hassan, R. A. Rosen and A. Schmidt-May, *JHEP* **1202**, 026 (2012) [arXiv:1109.3230 [hep-th]].
- [24] G. D’Amico, G. Gabadadze, L. Hui and D. Pirtskhalava, *Phys. Rev. D* **87**, 064037 (2013) [arXiv:1206.4253 [hep-th]].
- [25] G. Gabadadze, R. Kimura and D. Pirtskhalava, *Phys. Rev. D* **91**, no. 12, 124024 (2015) [arXiv:1412.8751 [hep-th]]. *Phys. Rev. D* **90** (2014) 2, 024029 [arXiv:1401.5403 [hep-th]].

- [26] Q. G. Huang, Y. S. Piao and S. Y. Zhou, Phys. Rev. D **86**, 124014 (2012) [arXiv:1206.5678 [hep-th]];  
A. De Felice and S. Mukohyama, Phys. Lett. B **728**, 622 (2014) [arXiv:1306.5502 [hep-th]];  
G. Gabadadze, K. Hinterbichler, J. Khoury, D. Pirtskhalava and M. Trodden, Phys. Rev. D **86**, 124004 (2012) [arXiv:1208.5773 [hep-th]];  
C. de Rham, A. Matas and A. J. Tolley, Class. Quant. Grav. **31**, 165004 (2014) [arXiv:1311.6485 [hep-th]];  
C. de Rham, A. Matas and A. J. Tolley, Class. Quant. Grav. **32**, no. 21, 215027 (2015) [arXiv:1505.00831 [hep-th]];  
C. de Rham and A. J. Tolley, Phys. Rev. D **92**, no. 2, 024024 (2015) [arXiv:1505.01450 [hep-th]];  
A. De Felice and S. Mukohyama, Phys. Lett. B **752**, 302 (2016) [arXiv:1506.01594 [hep-th]].
- [27] T. W. B. Kibble, Commun. Math. Phys. **64**, 73 (1978).  
T. W. B. Kibble and S. Randjbar-Daemi, J. Phys. A **13**, 141 (1980).  
V. P. Frolov and G. A. Vilkovisky, in Quantum Gravity, proceedings of the Second Moscow Seminar, edited by M.A. Markov and P. C. West (Plenum, New York, 1983).  
A. Vilkovisky, in Quantum Theory of Gravity , edited by S.M. Christensen (Adam Hilger, Bristol, 1984).
- [28] R. D. Jordan, Phys. Rev. D **33**, 444 (1986).
- [29] D. Anselmi, JHEP **0701**, 062 (2007) [hep-th/0605205].
- [30] C. Deffayet, J. Mourad and G. Zahariade, JHEP **1303**, 086 (2013) [arXiv:1208.4493 [gr-qc]]; JCAP **1301**, 032 (2013) [arXiv:1207.6338 [hep-th]].
- [31] M. Mirbabayi, Phys. Rev. D **86**, 084006 (2012) [arXiv:1112.1435 [hep-th]].
- [32] T. Kugo and N. Ohta, PTEP **2014** (2014) 043B04 [arXiv:1401.3873 [hep-th]].
- [33] X. Gao, T. Kobayashi, M. Yamaguchi and D. Yoshida, Phys. Rev. D **90**, no. 12, 124073 (2014) [arXiv:1409.3074 [gr-qc]].
- [34] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. **305**, 96 (2003) [hep-th/0210184].
- [35] M. A. Luty, M. Porrati and R. Rattazzi, JHEP **0309**, 029 (2003) [hep-th/0303116];  
A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79**, 064036 (2009) [arXiv:0811.2197 [hep-th]].
- [36] A. I. Vainshtein, Phys. Lett. B **39**, 393 (1972).
- [37] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, Phys. Rev. D **65**, 044026 (2002) [hep-th/0106001].

- [38] E. Babichev and C. Deffayet, *Class. Quant. Grav.* **30**, 184001 (2013) [arXiv:1304.7240 [gr-qc]].
- [39] J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995) [astro-ph/9505060].
- [40] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999) [hep-th/9906064].
- [41] K. Hinterbichler and R. A. Rosen, *JHEP* **1207**, 047 (2012) [arXiv:1203.5783 [hep-th]].
- [42] G. Gabadadze, K. Hinterbichler, D. Pirtskhalava and Y. Shang, *Phys. Rev. D* **88**, no. 8, 084003 (2013) [arXiv:1307.2245].
- [43] K. Hinterbichler, J. Stokes and M. Trodden, *Phys. Rev. D* **92**, no. 6, 065025 (2015) [arXiv:1505.05513 [hep-th]].
- [44] D. Z. Freedman, K. Johnson and J. I. Latorre, *Nucl. Phys. B* **371**, 353 (1992).
- [45] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998) [hep-th/9802150];  
D. Z. Freedman, S. D. Mathur, A. Matusis and L. Rastelli, *Nucl. Phys. B* **546**, 96 (1999) [hep-th/9804058];  
W. Mueck and K. S. Viswanathan, *Phys. Rev. D* **58**, 041901 (1998) [hep-th/9804035].
- [46] S. de Haro, S. N. Solodukhin and K. Skenderis, *Commun. Math. Phys.* **217**, 595 (2001) [hep-th/0002230];  
M. Bianchi, D. Z. Freedman and K. Skenderis, *Nucl. Phys. B* **631**, 159 (2002) [hep-th/0112119];  
K. Skenderis, *Class. Quant. Grav.* **19**, 5849 (2002) [hep-th/0209067].
- [47] W. Mueck and K. S. Viswanathan, hep-th/9810151.
- [48] G. E. Arutyunov and S. A. Frolov, *Nucl. Phys. B* **544**, 576 (1999) [hep-th/9806216].